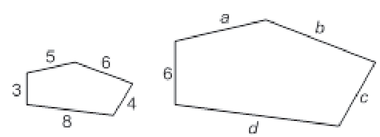
Unit Title	3-D Required		**Length of Unit**	5 weeks
Focusing Lens(es)	Relationships Visualization	**Standards and Grade Level Expectations Addressed in this Unit**	MA10-GR.7-S.4-GLE.1 MA10-GR.7-S.4-GLE.2	
Inquiry Questions (Engaging-Debatable):	• Why is pi an important number? (MA10-GR.7-S.4-GLE.2-IQ.8) • How many two-dimensional shapes can you make by slicing a three-dimensional object?			
Unit Strands	Geometry			
Concepts	Circumference, area, circle, diameter, π, ratio, radius, slice, three-dimensional figures, two-dimensional figures, scale factor, magnification, zoom level, scale drawings, characteristics, drawing, tools (rulers, protractors, compasses), complementary, supplementary, adjacent, vertical, angles, indirect measurement, additive property, area, volume, decomposition, composition			

Generalizations **My students will Understand that…**	**Guiding Questions** **Factual**	**Conceptual**
Mathematicians recognize the special relationship between the diameter and circumference of a circle as the ratio called π, and utilize this relationship to calculate the area, circumference, diameter or radius of a circle. (MA10-GR.7-S.4-GLE.2-EO.a, b)	What is the radius? What is the formula for finding the circumference of a circle? What is the formula for finding the area of a circle? What is π?	How are the circumference and diameter of a circle related? How does the derivation of the formula for the area of a circle rely on both the circumference and radius of the circle?
Slicing three-dimensional figures results in two-dimensional figures (MA10-GR.7-S.4-GLE.1-EO.a.iv)	What types of two-dimensional figures can be created when slicing a cone?	How does slicing a 3-D shape parallel to the base differ from slicing the same 3-D shape diagonal to the base?
Mathematicians represent scale factor in terms of magnification or zoom level. (MA10-GR.7-S.4-GLE.1-EO.a.i)	How does scale factor affect length, perimeter, angle measure, area, and volume? (MA10-GR.7-S.4-GLE.1-IQ.3)	Why is the scale factor for side lengths and perimeters different from the one for areas?
Mathematicians draw geometric figure using rulers, protractors, and compasses with precision (MA10-GR.7-S.4-GLE.1-EO.a.ii, a.iii)	How is sketching different from drawing? When drawing triangles, when do provided characteristics lead to no triangle, exactly one or more than one possible triangle? Is there a geometric figure for any given set of attributes? (MA10-GR.7-S.4-GLE.1-IQ.1)	Why are rulers, protractors and compasses necessary when drawing shapes?

Angle relationships such as complementary, supplementary, adjacent and vertical angles provide mathematicians an indirect means to solve for unknown angles in a figure (MA10-GR.7-S.4-GLE.2-EO.c)	What are complementary angles? What are supplementary angles? What are adjacent angles? What are vertical angles? How do line relationships affect angle relationships?	How can you indirectly determine the measurement of an unknown angle formed by two intersecting lines? How can geometric relationships among lines and angles be generalized, described, and quantified? (MA10-GR.7-S.4-GLE.2-IQ.1)
The additive property of area and volume provides a means for deriving equations to find the surface area and volume of two -and three-dimensional objects (MA10-GR.7-S.4-GLE.2-EO.d)	What are examples of familiar shapes that are helpful to recognize within larger objects when trying to find volumes or surface areas? What do surface area and volume tell about an object? (MA10-GR.7-S.4-GLE.2-IQ.6)	Why area and volume both have additive properties of composition and decomposition? How can two shapes have the same volume but different surface areas and vice versa? (MA10-GR.7-S.4-GLE.2-IQ.2, 3)

Key Knowledge and Skills: **My students will…**	*What students will know and be able to do are so closely linked in the concept-based discipline of mathematics. Therefore, in the mathematics samples what students should know and do are combined.*

- **Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. (MA10-GR.7-S.4-GLE.1-EO.a.i)**
  - Students recognize scale drawings as dilations, and they have experience working with scaled objects such as building models, reading maps, and resizing photographs on the computer. They may, therefore, think of a scale factor in terms of magnification or zoom level. **(CC.7.G.1)**
  - Students use their abilities to work with ratio relationships to find missing values in scale drawing problems with well-defined measurements. **(CC.7.G.1)**
  - For example, using the following scale drawing to find the missing side lengths:



(CC.7.G.1)

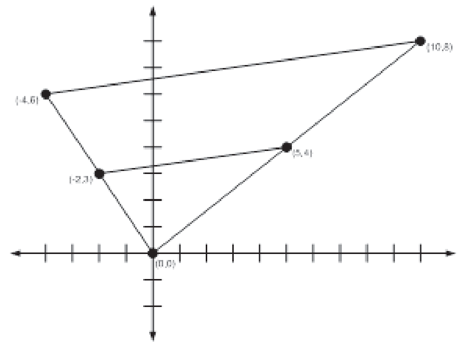
  - Students approach this problem several ways. One approach is to determine the scale factor between the two diagrams and write an equation representing the relationship. This is done by solving for the constant of proportionality, k, in the equation $6 = k \cdot 3$. Once the scale factor of 2 is determined, students then setup and solve equations for each of the missing values. For instance, $a = 2 \cdot 5$ or 10. **(CC.7.G.1)**
  - Another approach is to set up a ratio table and fill in the missing values by using strategies from prior work with ratio relationships, such as ratio boxes. Students create a table similar to the one below, and solve for each variable by creating a ratio box using the first row and the row containing that variable: **(CC.7.G.1)**

3	6
5	*a*
6	*b*
4	*c*
8	*d*

(CC.7.G.1)

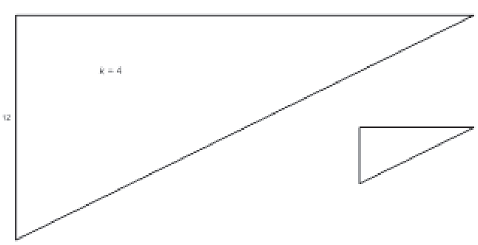
  - For geometric figures in the coordinate plane with one vertex at the origin, students use the scale factor and equation as above for both the *x* and *y* coordinates of each vertex in order to create a scale drawing. **(CC.7.G.1)**

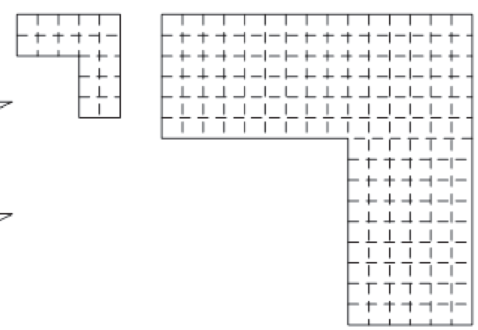
- For example, given a triangle with vertices at (0, 0), (-2, 3) and (5, 4) create a scale drawing with scale factor $k = 2$. Students multiply both coordinates of each vertex by 2 and find that the new triangle will have vertices at (0, 0), (-4, 6) and (10, 8), and they graph the two on the same plane as shown below:



(CC.7.G.1)

- Students measure lengths and calculate perimeters, areas, and volumes using formulas and compositions of figures. They recognize that because perimeter is a length measurement, the perimeters of similar figures are in the same proportion as the respective side lengths. Students also begin to notice that the areas of common geometric figures in scale drawings do not maintain the same ratio as the side lengths between figures. However, a pattern in the ratio of the areas of two figures in a scale drawing can be seen through exploration with several different common shapes using area formulas and drawings on grid paper where the counting of unit squares can be utilized. It is important to allow students to explore this relationship with a variety of shapes to convince them that the pattern holds. **(CC.7.G.1)**
  - *Note to teachers*: When asked to give the scale factor for comparing areas, students should be able to justify squaring the scale factor for length and not just repeating the fact from memory. **(CC.7.G.1)**
- Consider the following pairs of scale drawings:

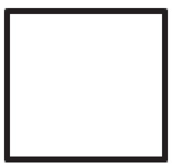




- Once given the formula, $A_1 = k^2 \cdot A_2$, students calculate the area of one figure in a scale drawing given the other area, or they determine the scale factor given both areas. **(CC.7.G.1)**

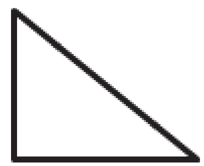
- **Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given condition, with focus on triangles from three measures of angles or sides, noting when the conditions determine a unique triangle, more than one triangle, or no triangle. (MA10-GR.7-S.4-GLE.1-EO.a.ii, a.iii)**
  - Students are familiar with geometric shapes and their attributes and are able to name and identify them. They are first able to sketch triangles, quadrilaterals and other geometric shapes freehand. In doing so, students pay attention to the differences in attributes and characteristics that distinguish different classifications of shapes. Often times these sketches are inaccurate in that they are not perfectly to scale and certain qualities of a shape may be exaggerated. **(CC.7.G.2)**
  - For instance, students offer the following drawings for specified shapes:

Square



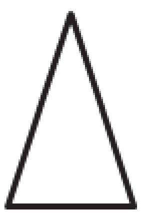
Rectangle

Right Triangle

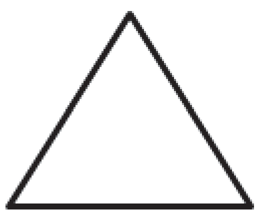


(CC.7.G.2)

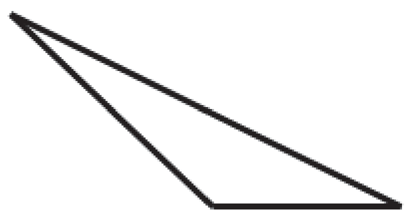
Isosceles Triangle



Equilateral Triangle



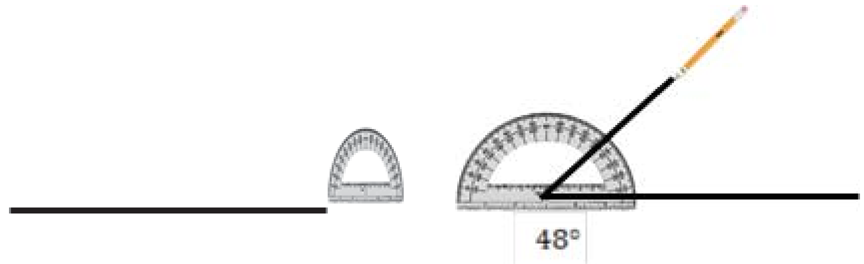
Obtuse Triangle



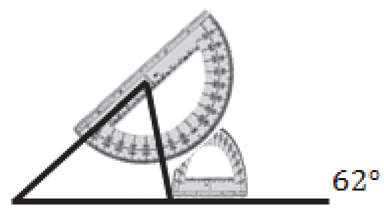
(CC.7.G.2)

  - Students move to more precise representations of geometric figures when they become proficient in their use of tools for drawing, particularly rulers, protractors and compasses. In doing so, drawings become more accurate and specific to levels beyond shape classifications. Students at this stage are capable of creating representations of figures with exact measurements. **(CC.7.G.2)**

- For example, students use a protractor to draw a triangle with angle measures of $48°, 62°\ and\ 70°$**(CC.7.G.2)**



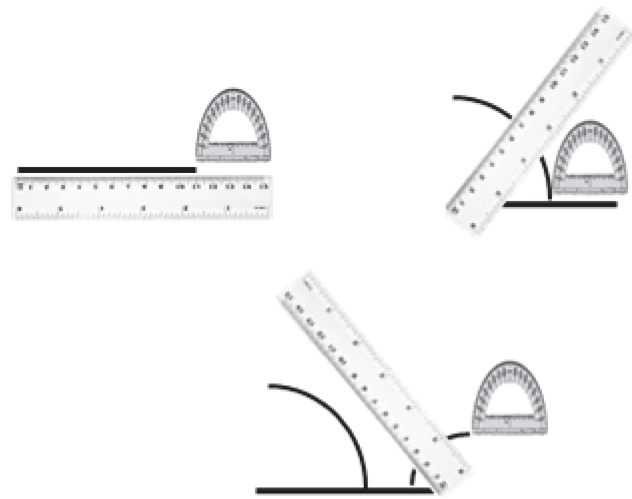
(CC.7.G.2)



- Students should be encouraged to experiment with constructing triangles given any three measurements (angles and/or sides) to see if it is possible construct a triangle that meets the given conditions – noting when it is not and when there is exactly one or more than one solution. Although this can be done one paper by hand with the use of tools, it is perhaps most easily done using technology such as dynamic geometry software. Such exercises will establish early reasoning for proofs of the triangle congruence and similarity theorems that will be encountered in students' high school coursework in geometry. It also forms a basis for formal constructions whereby only a compass and straightedge are used. **(CC.7.G.2)**
- Each description below could quickly and easily be imitated on dynamic geometry software. Consider the following three sets of side length (*S*) and angle (*A*) conditions for triangles: **(CC.7.G.2)**

1) $S = 11$ cm, $S = 7$ cm, $S = 2$ cm

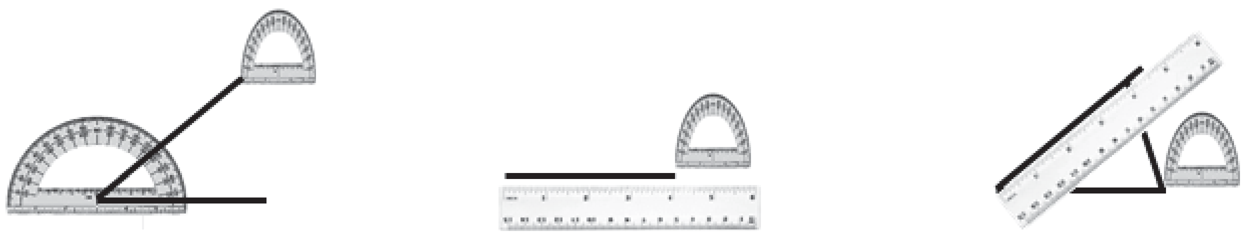
- Using a ruler, students measure and draw one side (it will not matter which side is drawn first but is easiest to see the result if the longest side is drawn first as shown below). They then hold the ruler such that 0 is at one end point of the first line segment and rotate the ruler about that point while holding their pencil at a point corresponding to one of the other side lengths. This process is repeated from the other endpoint of the first segment using the third side length. Here, the arcs do not cross and thus there is no such possible triangle. **(CC.7.G.2)**



(CC.7.G.2)

2) $s = 4\ in, A = 40°, S = 4\ in$

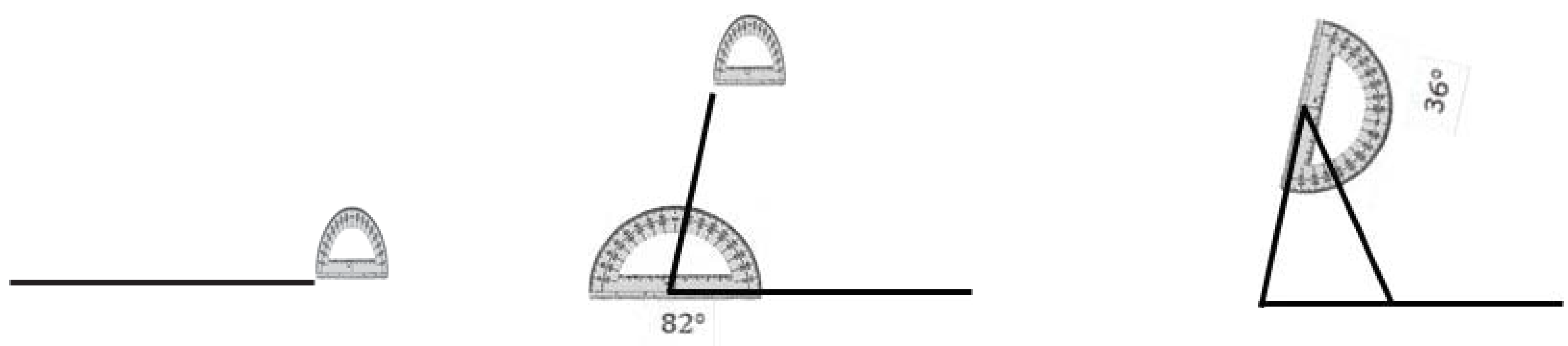
- Using a ruler and protractor, students measure and draw one side and then extend a ray from one endpoint of that side at the given angle. The second side is measured and the non-shared endpoint is connected to the other endpoint of the original side. It is clear from these steps that the resulting triangle is unique and the only possible triangle. **(CC.7.G.2)**



(CC.7.G.2)

3) $A = 82°, A = 36°, A = 62°$

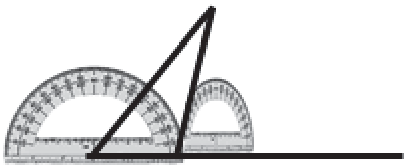
- Using a ruler and a protractor, students draw an arbitrary line and extend a ray from one endpoint at one of the given angles. It then becomes obvious that the protractor could be placed along that ray anywhere and a line segment can be drawn at a second given angle that will intersect the original line (or an extension of it). Therefore, there are infinitely many possible triangles. Students notice that all of these possible triangles are merely scale drawings of one another and could also be generated as such. **(CC.7.G.2)**

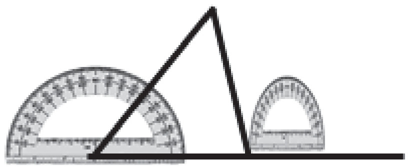


(CC.7.G.2)

- Students recognize that when a set of given measurements for a type of figure define one unique figure, then all such figures constructed that have those measurements will be congruent. Students recognize when a set of given measurements for a type of figure define more than one possible figure, or when no figure meets the conditions. **(CC.7.G.2)**

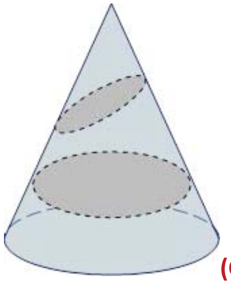
- For example, students discover that a triangle with side lengths of 3, 5, and 9 is not possible. Likewise, when constructing a triangle with an angle of and then 52° consecutive side lengths of 8 and 6, students recognize that there are two possible triangles that meet the description, as shown below:





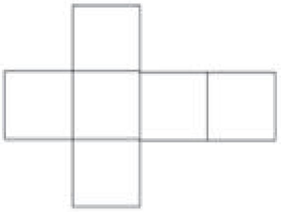
(CC.7.G.2)

- **Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids. (MA10-GR.7-S.4-GLE.1-EO.a.iv)**
  - In anticipation of high school geometry (e.g., conic sections) and Calculus, students visualize the 2-D figures that result from slicing 3-D figures using straight and diagonal cuts. For example, they understand that a 3-D cone when sliced parallel to the horizon creates a circle and when sliced diagonally creates an ellipse. **(CC.7.G.3)**

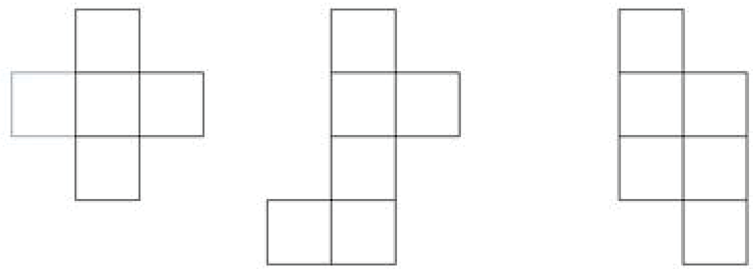


(CC.7.G.2)

- **Know the formulas for the area and circumference of a circle and use them to solve problems (MA10-GR.7-S.4-GLE.2-EO.a)**
  - Building on previous exploration of three-dimensional shapes, students work with 3-D models (excluding cones, cylinders, and spheres) to identify faces, edges, and vertices. As their spatial reasoning develops, students recognize that certain two-dimensional representations (nets) could be cut and folded to form a particular three-dimensional shape. **(CC.7.G.4)**
  - In this standard, students compose 2-D shapes to form a 3-D shape and decompose 3-D shapes to its 2-D components. For example, students may recognize that the cube can be created by folding 6 squares, and thus being introduced to the idea of “nets” for producing 3D shapes: **(CC.7.G.4)**

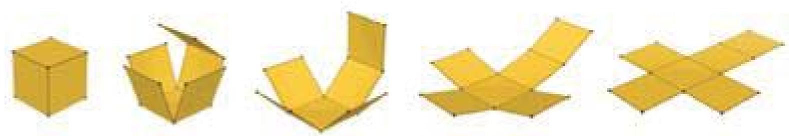


- Students begin to identify nets that can be assembled to form 3-D shapes. For example, which of the following nets makes a cube? **(CC.7.G.4)**



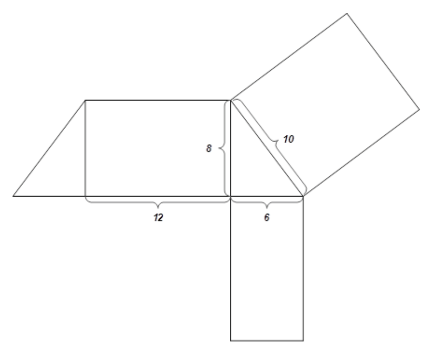
(CC.7.G.4)

- Dynamic geometry software may be used to help students to visualize the composition and decomposition of 3-D shapes: **(CC.7.G.4)**



(CC.7.G.4)

- Students also use the nets to determine the surface area of the 3-D figures which are made up of triangular and rectangular faces. For example, "The city of Raleigh is erecting a triangular prism sculpture. The base frame of the sculpture is a right-angle triangle of base 6 meters, height 8 meters and the longest side 10 meters. The sculpture stands tall at 12 meters. How much area of thick aluminum sheet is needed to cover the sculpture?" **(CC.7.G.4)**

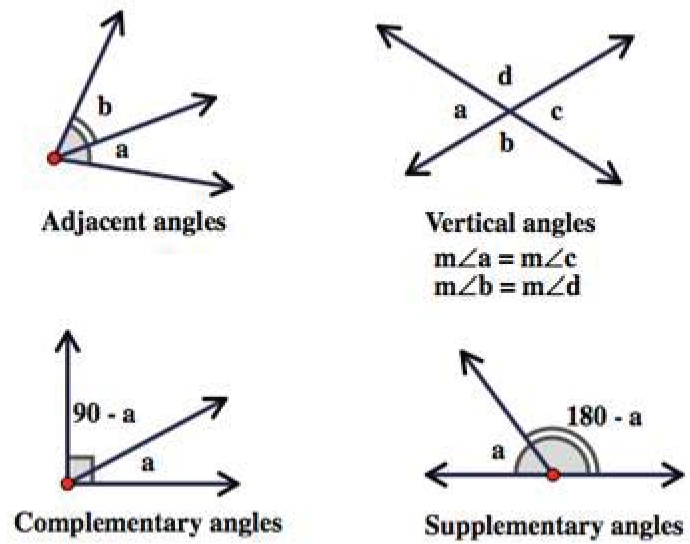


(CC.7.G.4)

- Students construct the net of the triangular prism to determine that the shapes of the net include two right-angle triangles of base 6 meter by height 8 meters and three rectangles of dimensions, 12 meters by 10 meters. 12 meters by 8 meters, and 12 meters by 6 meters. They recognize that the triangular face at the bottom of the prism is excluded and apply the area formulae of triangle and to calculate the total area of the other faces. **(CC.7.G.4)**

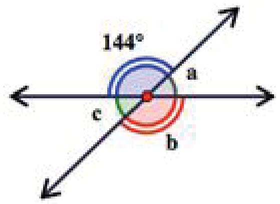
- **Give an informal derivation of the relationship between the circumference and area of a circle. (MA10-GR.7-S.4-GLE.2-EO.b)**
  - Tasks require students to identify or produce a logical conclusion about the relationship between the circumference and the area of a circle, e.g., that given three circles with areas $A1 > A2 > A3$ the circumferences satisfy $C1 > C2 > C3$. **PARCC**

- **Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. (MA10-GR.7-S.4-GLE.2-EO.c)**
  - Students recognize that:
    - *Complementary angles* are the angles whose measures add to 90° and can be composed to make right angles. **(CC.7.G.5)**
    - *Supplementary angles* are angles whose measures add to 180° and can be composed to make straight angles. **(CC.7.G.5)**
    - *Adjacent angles* are angles that share a vertex and a common ray between the two other rays, such that the angles do not overlap. In other words, adjacent angles are side by side. **(CC.7.G.5)**
    - *Vertical (or opposite) angles* are angles formed by two intersecting lines/segments/rays and that are not adjacent. These angles are equal in measure (which follows from the definition of supplementary angles and can be discovered in the Example 1 below). **(CC.7.G.5)**



(CC.7.G.5)

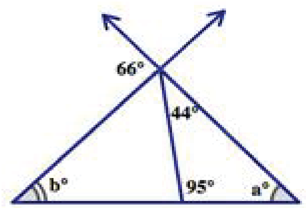
  - Students solve simple one-step word problems for an unknown angle. These can be of the types: **(CC.7.G.5)**
    - "The measure of an angle is 34°. What is the measure of a complementary angle?" or, **(CC.7.G.5)**
    - "The measure of an angle is 123°. What is the measure of a supplementary angle?" **(CC.7.G.5)**
  - Students also write and solve simple equations for an unknown angle, in a figure, in multi-step problems. For example, students find the unknown angles in the following figures: **(CC.7.G.5)**
    - <u>Example 1:</u> Find the measures of the missing angles *a*, *b*, and *c*.



(CC.7.G.5)

Angle *a* is supplementary to angle 144° so, *a* = 180° - 144° = 36°.
Angle *b* is supplementary to *a* so, *b* = 180° - 36° = 144°.
Angle *c* is vertical to angle *a*, so *c* = *a*, and if *a* = 36° then *c* = 36°.

(CC.7.G.5)



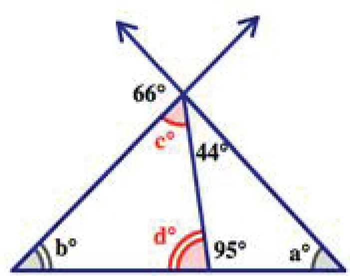
(CC.7.G.5)

- Example 2: Find the measure of missing angles *a*, and *b*. **(CC.7.G.5)**

a + 44° + 95° = 180° (the sum of the angles in a triangle is 180°)
a = 180° - 44° - 95° = 41°

(CC.7.G.5)

Then, you can find angle *c*, which is supplementary to the sum of angles measuring 66° and 44°.
c = 180° - 44° - 66°
c = 70°



(CC.7.G.5)

And then angle d which is supplementary of angle 95°.

d = 180°- 95°
d = 85°
(CC.7.G.5)
Finally, for angle b:
b = 180 - c – d
b = 180 -70 -85
b = 25°
(CC.7.G.5)

- *Note to teachers:* Please note that these are not the only solutions possible but on the contrary that are different ways to find those missing angles.

- **Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (MA10-GR.7-S.4-GLE.2-EO.d)**
  - Eventually, students decompose 2-D areas and 3-D structures (that can be decomposed) into their parts and measure each of the parts to determine the shape's area or object's volume. **(CC.7.G.6)**
  - It is important that students fully understand the additive property of area and volume along with how to recognize both attributes on a three-dimensional object. For example, in order to find the surface area of a cube, students must recognize that there are six areas on the cube and that the surface area can be found by adding these six areas. **(CC.7.G.6)**
  - Students should be familiar with surface area formulas for 2-D objects in order to derive equations to find the surface area of 3-D objects that are composed of triangles, quadrilaterals, polygons, cubes, and right prisms. For example, they should be able to decompose and find the area of triangles and rectangles in order to find the surface area of a right prism. **(CC.7.G.6)**

Critical Language: includes the Academic and Technical vocabulary, semantics, and discourse which are particular to and necessary for accessing a given discipline.
EXAMPLE: A student in Language Arts can demonstrate the ability to apply and comprehend critical language through the following statement: *"Mark Twain exposes the hypocrisy of slavery through the use of satire."*

A student in ______________ can demonstrate the ability to apply and comprehend critical language through the following statement(s):	*The area of a circle is derived by cutting the circle like a pizza into successively smaller slices and rearranging to form a parallelogram with a base that is half the circumference and a height of the radius.*
Academic Vocabulary:	Solve, draw, freehand, ruler, protractor, triangle, area, circle, angles, polygons cubes, slice, three-dimensional figures, two-dimensional figures, scale factor, magnification, zoom level, scale drawings, characteristics, volume, derive, parallelogram
Technical Vocabulary:	Circumference, diameter, π, ratio, radius, drawing, tools (rulers, protractors, compasses), complementary angles, supplementary angles, adjacent angles, vertical angles, indirect measurement, additive property, decomposition, composition, congruent quadrilateral, right prisms